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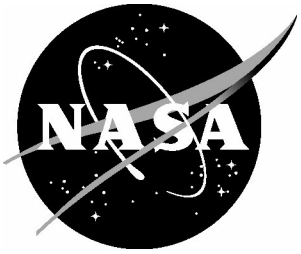

Scale-free Graphs for General Aviation Flight Schedules

Rex K. Kincaid
College of William and Mary, Williamsburg, Virginia

November 2003

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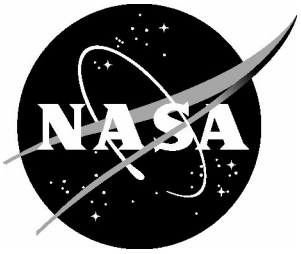

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Rex K. Kincaid
College of William and Mary, Williamsburg, Virginia

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23681-2199

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Abstract

In the late 1990s a number of researchers noticed that networks in biology, sociology, and telecommunications exhibited similar characteristics unlike standard random networks. In particular, they found that the cummulative degree distributions of these graphs followed a power law rather than a binomial distribution and that their clustering coefficients tended to a nonzero constant as the number of nodes, n, became large rather than $O(1/n)$. Moreover, these networks shared an important property with traditional random graphs—as n becomes large the average shortest path length scales with log n. This latter property has been coined the small-world property. When taken together these three properties—small-world, power law, and constant clustering coefficient—describe what are now most commonly referred to as scale-free networks. Since 1997 at least six books and over 400 articles have been written about scale-free networks. In this manuscript an overview of the salient characteristics of scale-free networks. Computational experience will be provided for two mechanisms that grow (dynamic) scale-free graphs. Additional computational experience will be given for constructing (static) scale-free graphs via a tabu search optimization approach. Finally, a discussion of potential applications to general aviation networks is given.

1 Background Information on Random Graphs

The birth of the mathematical study of random graphs is generally attributed to the seminal work of Erdos and Renyi [29, 30]. Since then a steady stream of books and research articles have studied these mathematical objects. A graph is a collection of n nodes (vertices) and edges (links, arcs, bonds) that connect the nodes. In a random graph we are given a probability p that a pair of nodes has an edge connecting them. Assuming independence, the expected number of edges in a random graphs is $p \cdot n(n-1)/2$. Typically these graphs are denoted G_p or $G_{n,p}$. There are a variety of interesting properties that have been discovered about random graphs. Three that will be central to our discussion are the average shortest path length, the degree distribution and the clustering or transitivity coefficient.

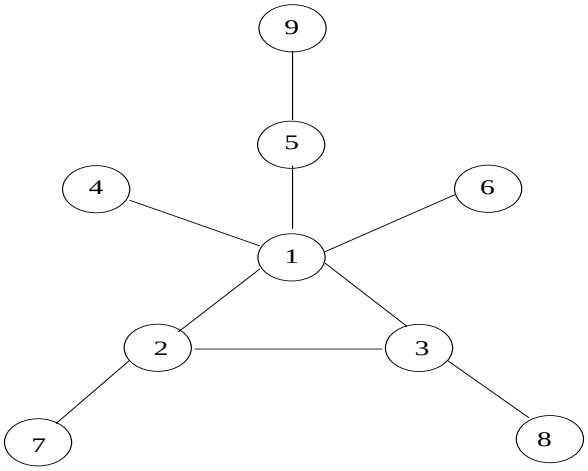

Fig. 1. A Graph with 9 nodes and 9 edges

The degree of a node in an undirected graph is the number of incident edges. For example, in Figure 1 the degree of node 1 is 5. The degree sequence is $\{1,2,3,5\}$ and the fraction of the nodes of each degree k are $p(k) = \{5/9, 1/9, 2/9, 1/9\}$ respectively. This latter set is called the degree distribution. For random graphs $p(k)$ is known to be binomially distributed and, as n approaches infinity, Poisson distributed. Often the cummulative distribution $P(k) = \{1, 4/9, 3/9, 1/9\}$ where $P(k)$ denotes the fraction of nodes with degree k or larger is of greater interest.

The shortest path distances for an undirected graph with no edge weights is the number of edges between nodes on these paths. In general, an $O(n^3)$ algorithm is needed to compute the shortest path distance matrix. The shortest path distance matrix for the graph in Figure 1 is

$$SPD = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2 \\ 1 & 0 & 1 & 2 & 2 & 2 & 1 & 1 & 3 \\ 1 & 1 & 0 & 2 & 2 & 2 & 1 & 1 & 3 \\ 1 & 2 & 2 & 0 & 2 & 2 & 3 & 3 & 3 \\ 1 & 2 & 2 & 2 & 0 & 2 & 3 & 3 & 1 \\ 1 & 2 & 2 & 2 & 2 & 0 & 3 & 3 & 3 \\ 2 & 1 & 2 & 3 & 3 & 3 & 0 & 3 & 4 \\ 2 & 1 & 1 & 3 & 3 & 3 & 3 & 0 & 4 \\ 2 & 3 & 3 & 3 & 1 & 3 & 4 & 4 & 0 \end{pmatrix}$$

There is one row and one column for each node. For random graphs it is known that the average shortest path distance scales with the *log* of the number of nodes. The third metric associated with random graphs of interest to us is the transitivity or clustering coefficient. The goal is to find a $(0,1)$ scale for keeping track of how many complete subgraphs on three nodes (triangles) are present. One way to do this is to compute

$$C^{(1)} = \frac{\text{[three times total number of triangles]}}{\text{[total number of connected three tuples of nodes]}.}$$

The three in the numerator means that triangles count as three distinct three tuples while the denominator is equivalent to counting the distinct paths of length two. A second popular method used to capture transitivity is to first compute for each node i

$$C_i = \frac{\text{[number of triangles in which node } i \text{ is incident]}}{\text{[number of three tuples of connected nodes centered on node } i\text{]}.}$$

Then we define $C^{(2)} = 1/n\sum_i C_i$. For the graph in Figure 1, $C^{(1)} = 1/5$ and $C^{(2)} = 11/90$. In general, both clustering coefficients tend to (average degree)$/(n-1)$ as n becomes large for G_p graphs. Hence, as n goes to infinity the clustering coefficient goes to zero.

In optimization modeling random graphs are a frequent mechanism for generating testbeds to compare the performance of competing algorithms. For example, in [42] two distinct classes of random graphs are generated to both tune the parameters of their implementation of simulated annealing for the graph partitioning problem and to compare its performance against other competing heuristics. The two classes of random graphs provided a set of repeatable experiments (both rely on a psuedorandom number generator and a seed value) and an easy way to test relatively large problem instances. Graphs with up to 1500 nodes were examined.

2 Properties of Scale-Free Graphs

In the late 1990s a number of researchers noticed that *real* graphs in biology, sociology, and telecommunications exhibited similar graph characteristics unlike standard random graphs. In particular, they found that the cummulative degree distributions followed a power law rather than a binomial distribution and that the clustering coefficients tended to a nonzero constant as n became large rather than $O(1/n)$. Moreover, these *real* graphs shared the property with random graphs that as n becomes large the average shortest path length scales with *log n*. This latter property has been coined the *small-world* property. When taken together these three properties—small-world, power law, and constant clustering coefficient—describe what are now most commonly referred to as scale-free graphs.

As a result of these observations an explosion of activity commenced. Since 1998 several books have been dedicated to the topic including the popular books *Linked* by Barabási [8], *Six Degrees* by Watts [64] and *Nexus* by Buchanan [17]. In addition, a number of excellent review articles exist including Newman [53], Strogatz [60], Albert and Barabási [3] and Dorogovtsev and Mendes [24]. These review articles contain hundreds of references. For example [53] contains 429 references. We note, as have many other authors, that graphs whose degree distributions follow a power law are not new. For example, Milgram's [51] work with acquaintance networks in the United States which led to his conclusion there were *six degrees of separation* amongst the individuals studied. However, the early references to graphs following power laws are small in number when compared to the current explosion in interest.

Table 1 provides a summary of essential characteristics of four *real* graphs. The interested reader is referred to Newman [53] for a more exhaustive table. Clearly the average shortest path lengths for the graphs in Table 1 grow slowly. For example, the value of 16.18 for the Altavista world wide

web graph means that Altavista web pages are on average 16 clicks away even though there are potentially 2.1 billion edges to traverse. If the average shortest path length scales logarithmically or slower with the number of nodes in the graph then it is called a *small-world* graph (see Watts [64]). The natural logarithms of the number of nodes for the graphs listed in Table 1 are 13.02, 19.13, 9.28, and 7.66 respectively. Consequently, all graphs listed in Table 1 exhibit the *small-world* property. However, this property alone does not capture scale-free graphs as traditional random graphs are also *small-world*.

Graph	Nodes	Edges	AvgDeg	AvgSPD	$C^{(1)}$	$C^{(2)}$	Ref
film actor	449,913	25,516,482	113.443	3.48	0.20	0.78	11,12
Altavista	203,549,046	2,130,000,000	10.46	16.18	—	—-	13
Internet	10,697	31,992	5.98	3.31	0.035	0.39	14,15
protein	2,115	2,240	2.12	6.80	0.072	0.071	16

Table 1. Basic statistics for a number of published graphs

We have already noted that the tail of the cummulative degree distributions for scale-free graphs follow a power law. This is easiest to see if $P(k)$ versus k is plotted on a log-log scale. If the tail is following a power law the graph of the tail will be a line on the log-log scale where the slope of the line is the exponent of the power function. If instead the tail is following an exponential distribution then a log-linear plot will be nearly linear. Figure 2 is a log-linear plot of a cummulative degree distribution with an exponential tail. The graph that gave rise to Figure 2 was generated in the following way. First, 1000 random (x,y) coordinates were generated in a 100 by 100 unit square. Next, the first three of these points was chosen as the initial set of nodes. Two edges were added by connecting node 1 to node 2 and node 2 to node 3. The average degree of this initial graph is $4/3$. Additional nodes were added one at a time. Edges were placed between the new node and existing nodes by selecting the nodes closest, with respect to Euclidean distance, to the new node. The number of new edges to be added is computed by taking the nearest integer of the average degree of the current graph. Real networks whose degree distributions display exponential tail behavior include the power grid of the Western United States (see [53] page 187). Figure 3 is a log-log plot of a cummulative degree distribution of a graph following a power law. As in the previous case the graph is generated by starting with the same three node two edge graph. Additional nodes are added one at a time. As before the number of edges to be added is computed as the nearest integer of the average degree of the current graph. The difference is that node connections are based upon roulette wheel selection. The roulette wheel is divided into sectors following the degree distribution of the current graph (ie. preferential attachment). Clearly Figure 3 is nearly linear and hence is associated with a scale-free graph. The codes used to generate both of these random graphs are provided in the Appendix.

Table 1 also records the average degree of each graph in column 4. This number does not provide degree distributional information but it is still a useful metric to record. The average degree is used to estimate the clustering coefficient when the number of nodes is large. Moreover, in the next section we will use the average degree as one of the objectives in a bi-objective approach to generate scale-free graphs. The optimization approach is in contrast to the preferential attachment description of scale-free graphs promoted by Barabási [8] and many others. Lastly, Table 1 records the two clustering coefficients in columns 6 and 7. It is impossible to verify whether or not these values are constant for these graphs as the number of nodes increases. An effective display of this property would be a plot of the clustering coefficient as the number nodes increases over time. Plots of this sort appear most often for scale-free graphs generated by some artificial mechanism that can be controlled. However, we can verify that the clustering coefficients in Table 1 are much larger than the expected value for a G_p graph. For a G_p graph the expected value of the clustering coefficient is the average degree divided by $n-1$. So, for example, if the film actor graph was similar to a G_p

graph then the expected clustering coefficient would be $113.443/449,913$ or $.000252$ which is three orders of magnitude smaller than either $C^{(1)} = 0.20$ or $C^{(2)} = 0.78$.

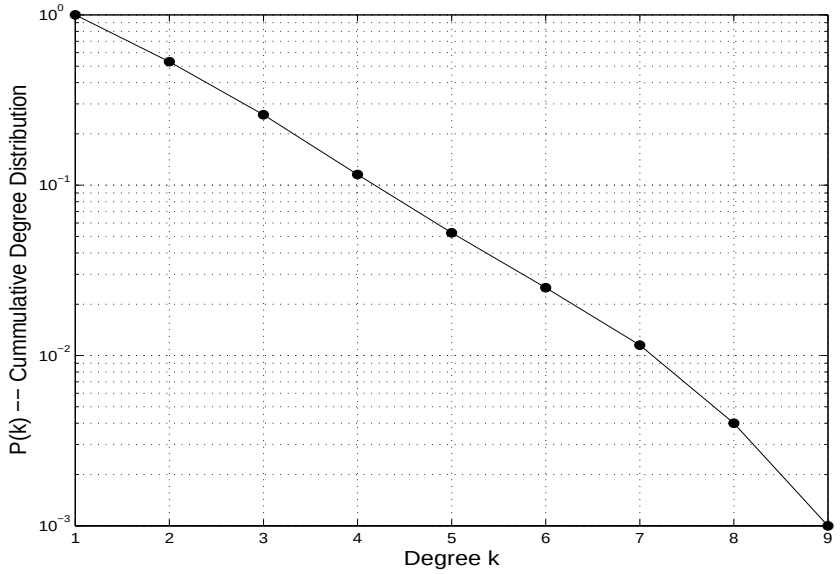

Fig. 2. 2000 nodes, degree = current average degree, edges closest Euclidean nodes

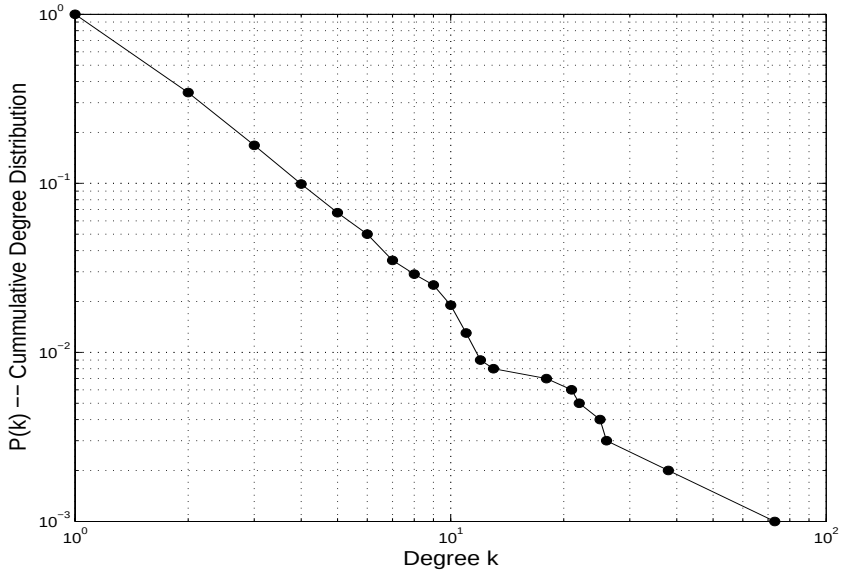

Fig. 3. 1000 nodes, edges added preferentially based on degree distribution

3 Optimization and Scale-Free Graphs

One of the captivating features of scale-free graphs is that they arise naturally without the aid of a human designer. Two intriguing questions are why and how? One partial answer to the *why* question

is error tolerance and communication. Communication between the nodes in a scale-free graph is carried out efficiently. One of the defining properties of scale-free graphs is that the length of the average shortest path is small. For example, in Table 1 we noted that in the Altavista graph with 2.1 billion edges the average shortest path length is 16.18. It is not clear, however, exactly how the entities traversing these graphs know how to find these paths. Kleinberg [44], in a well referenced article, appears to be one of the first to study the path decision problem in scale-free graphs. He was able to characterize a class of scale-free graphs, similar to those studied by Watts and Strogatz [63], for which a decentralized algorithm is able to find shortest paths with high probability. Although Milgram [51] did not directly address this question his conclusions would not have been possible unless the human subjects involved had been able to solve their own shortest path problems. There is also a downside to the existence of such effective communication. The objects represented by scale-free graphs are susceptible to virus transmission. If all of the 200 million nodes in the Altavista graph are about 16 clicks away then it is easy to see why computer viruses are transmitted so quickly.

Error tolerance is perhaps a less obvious criterion for the *why* question but is critical to the survival of *real* graphs. A variety of authors have noted that scale-free graphs are highly resistant to random edge and/or node failures. See, for example, [2], [21] and [13]. Scale-free graphs are, however, powerless against targeted node failures. That is, if an intelligent entity takes down the nodes of highest degree scale-free networks disconnect quickly. In response to this latter weakness several researchers have studied ways to strengthen scale-free graphs (see [18] and [57]).

The *how* question for scale-free graphs is still far from resolved. Initially the focus was on preferential attachment models which is how Figure 3 was generated. However, this left many researchers unconvinced and has led to bi-objective optimization models as well as dynamical systems models [39] and statistical mechanics models [3] A few authors have used some of the metrics listed in Table 1 as their objectives in a bi-objective approach to generate scale-free graphs. For example, Ferrer-Cancho and Solé [33] minimize a linear combination of average shortest path distance and the total number of edges in the graph. They show that, by varying the objective function weights, the resulting graphs vary from trees to scale-free graphs to a star graph (one node of degree $n-1$ and all other nodes of degree 1). A similar result is obtained by Mathias and Gopal [50]. Solé et al [59] provide a discussion of evolving biological systems and the role of optimization in that context. In addition, they point out that preferential attachment does not provide an adequate explanation for the large clustering coefficients observed in real networks. They believe that clustering is a side effect of optimization. They conjecture that reliable communication and cost minimizing shapes are the organizing principles behind scale-free graphs. Following [33] Figure 4 is the log-log plot of the cummulative distribution of a scale-free graph found by minimizing $(\lambda \cdot \text{total number edges} + (1-\lambda) \cdot \text{average number of path edges})$ with $\lambda = 0.50$ via a tabu search heuristic. In contrast if we replace the total number of edges objective with average degree we can achieve the same result. Figures 5, 6 and 7 provide an example of a scale-free graph that arises by minimizing $(\lambda \cdot \text{average degree} + (1-\lambda) \cdot \text{average number of path edges})$ with $\lambda = 0.65$. As before the minimization was done via a tabu search strategy. Figure 5 illustrates the actual graph while figure 6 gives the cummulative degree distribution. Figure 6 is roughly linear indicating that the graph in Figure 5 is scale-free. Figure 7 is a histogram of the degree distribution of the graph in Figure 5.

Tabu search is a metaheuristic strategy that can be used to control a variety of local search heuristics. Tabu search seeks to exploit historical information gathered during the local search phase so that the search will not remain stalled at a local optima. There are a variety of mechanisms developed to avoid local optima including tabu lists, recency and frequency based diversification schemes, and the detection of basins of attraction. Although the roots of tabu search are present in a variety of early work in artificial intelligence and operations research the seminal paper is Glover [34]. Glover [35] and Glover and Laguna [36] provide a comprehensive list of techniques and applications associated with tabu search specifically and adaptive memory programming generally.

A plain vanilla tabu search algorithm was developed to minimize a bi-objective based on any

two of the following five objectives: average degree, total number of edges, average shortest path distance, average shortest number of edges (hop length), and average shortest Euclidean distance. The user must provide the adjacency matrix of the initial graph, the objectives to be considered, the value of λ for the convex combination of the two objectives, how long a move is to remain tabu, and the maximum number of iterations allowed. The neighborhood of a solution (available moves) is all $O(n^2)$ bit flips in the adjacency matrix. Since we are considering undirected graphs the adjacency matrix is symmetric. If the (i, j) entry is a 1 then there is an edge between nodes i and j and so the (j, i) entry must also be 1. The most improving move available at each iteration is selected. A move is made unavailable (tabu) for a pre-determined number of iterations. In subsequent iterations the inverse move is disallowed so as to avoid (hopefully) a return to a previously observed solution. We maintained two tabu lists, one for the (i, j) bit flip moves and one for each row of the adjacency matrix. The idea behind the row tabu list is to avoid focusing the search on a particular node. For graphs with more than 50 nodes the computation of the average shortest hop length (or average shortest path length) is expensive—$O(n^3)$—to compute. For the complete $O(n^2)$ neighborhood this would result in $O(n^5)$ work per iteration. Consequently, we chose to examine a randomly chosen subset of the $O(n^2)$ moves. Typically we examined about 20 percent of the neighborhood until progress slowed and we are in the basin of attraction of a locally (hopefully globally) optimal solution. At this point we switched to a complete neighborhood search for the last n iterations. Tabu search is by definition a heuristic so there is no guarantee that the plots in Figure 4 and 5 are for the globally optimal solutions.

We experimented with a number of objective function combinations. But only two combinations—the average degree and average hop length as well as the total number of edges and average hop length—resulted in scale-free graphs. We had hoped that the average shortest Euclidean distance would have resulted in a scale-free graphs since it is much more efficient to compute and update. In addition, we found that the scale-free property was sensitive to our choice of λ. Consequently, we are not convinced that such an optimization procedure accurately depicts what *real* scale-free networks are doing when they organize themselves. There is also no reason to suspect that only two objectives are involved and that these objectives must be in linear combination. An analogous situation arises in multiobjective decision theory. One approach is to ask decision makers a series of questions to enable the modeler to uncover the utility function of the decision maker. The utility function provides a mathematics mapping of the competing objectives into a single objective. Utility functions are difficult to determine and assumptions of linearity are often difficult to justify. The interested reader is referred to Keeney and Raiffa [58] for more information about utility theory and decision making.

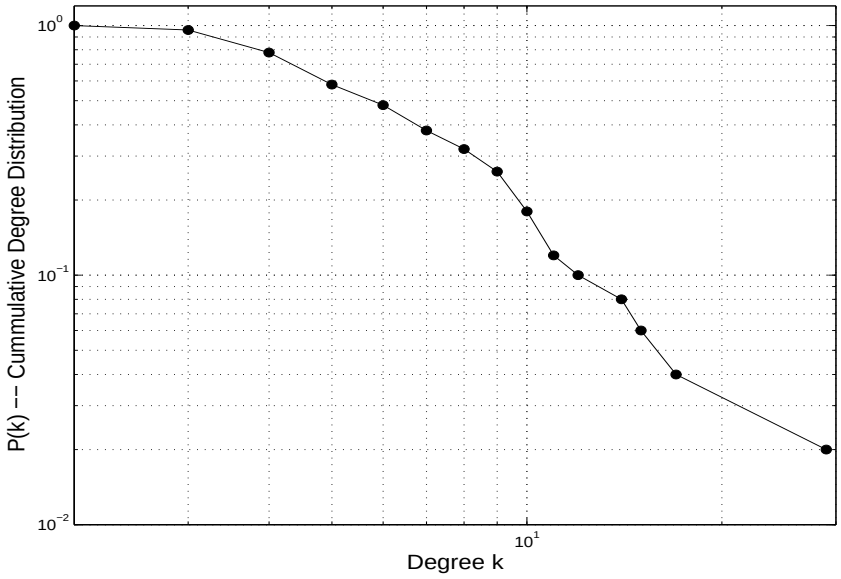

Fig. 4. Cummulative Degree Distribution: 50 nodes, $\lambda = 0.50$

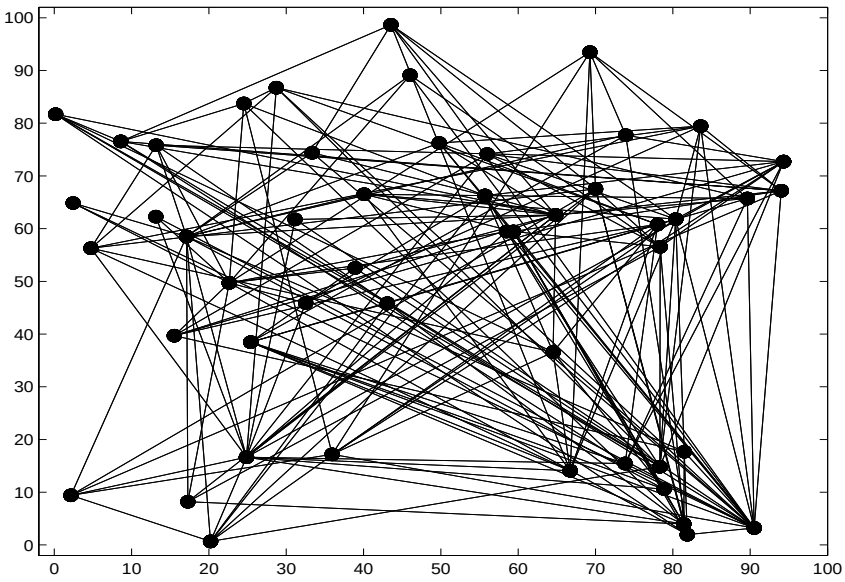

Fig. 5. Tabu Search result: 50 nodes, $\lambda = 0.65$

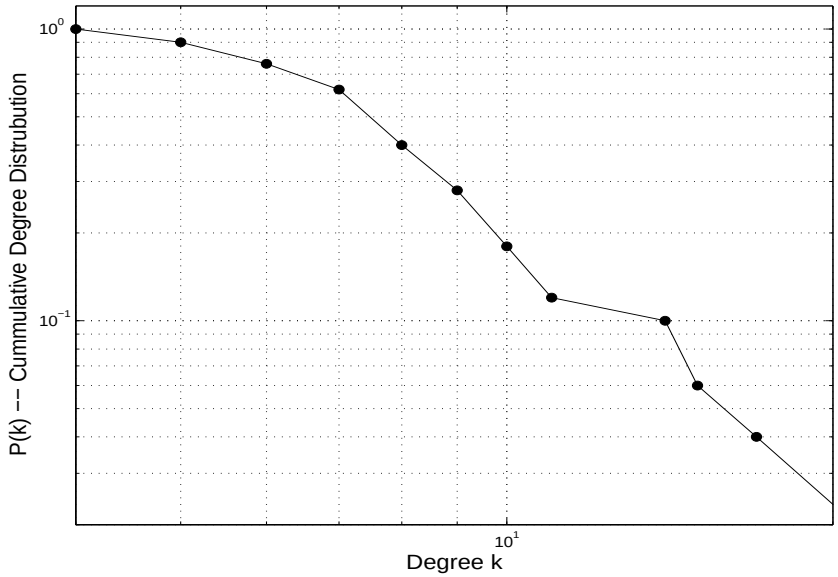

Fig. 6. Cummulative Degree Distribution: 50 nodes, $\lambda = 0.65$

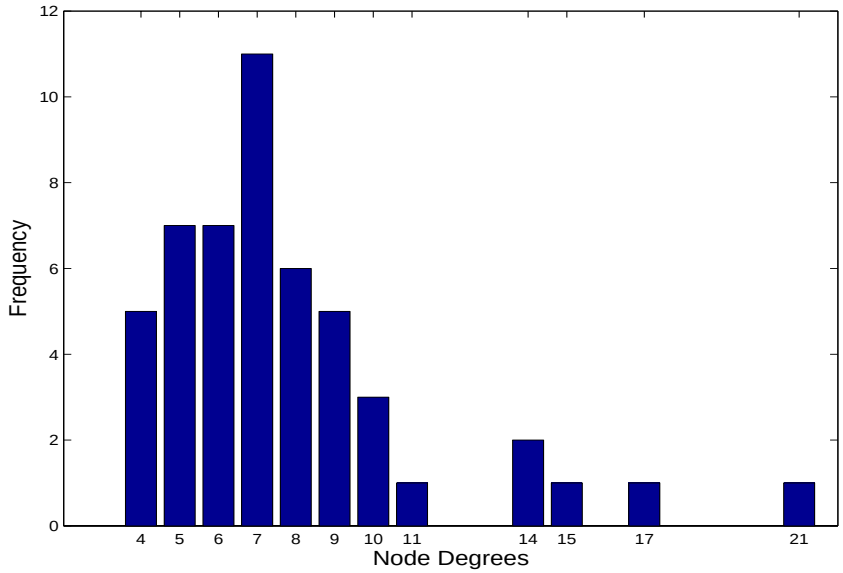

Fig. 7. Degree Distribution: 50 nodes, $\lambda = 0.65$

4 Graphs for General Aviation

The remaining question is what, if anything, can scale-free graphs be used to model in general aviation? To begin to answer this question we refer to two network design analyses. The first, Lederer and Nambimadom [47], compare the performance of four different network types–hub and spoke, direct, complete tour and a collection of subtours. Several simplifying assumptions are made. One assumption is that all *n* cities to lie equidistant from each other on a circle of fixed radius. In addition the demand for service is assumed to be identical for all origin-destination node pairs. The

hub lies at the center of the circle and is not allowed to be a destination. It should be clear how the hub and spoke network as well as the direct network routes are implemented. The complete tour routing does not utilize the hub at all. Instead it is a traversal of the n cities in one cycle. A collection of subtours partitions the n cities into k subcollections. Each subcollection is serviced by beginning at the hub and forming a cycle that includes all the cities in the subcollection. The performance measure is to maximize profit which is met by minimizing the sum of airline and passengers' cost subject to a fixed demand for service.

Lederer and Nambimadom [47] found that the optimal network choice was sensitive to three problem parameters—demand, distance between cities, and number of cities. If either demand or distance is small then a direct network is best, if either distance or demand is large than a complete tour is best. Intermediate values led to the other two networks. If the number of cities is small a direct network is best. If the number of cities is large a hub and spoke network wins. Intermediate values for the number of cities led to the other networks as best. In addition, they noted that the time built into a schedule to protect against delays is smaller for direct networks and that the direct networks were also the most reliable (e.g. Southwest Airlines). The authors also found that if fixed and variable costs have constant returns to scale then a direct network is optimal under the modeling assumptions. The scale-free community has also considered ring lattices. Watts and Strogatz [63] began the work in this direction. Figure 8 displays such a graph. The graphs are characterized by the degree of each node (all are identical initially) and a probability that an edge will be *rewired* to create a shortcut across the circle. Since these graphs have been well studied in the scale-free literature they provide a natural avenue for further study following Lederer's approach.

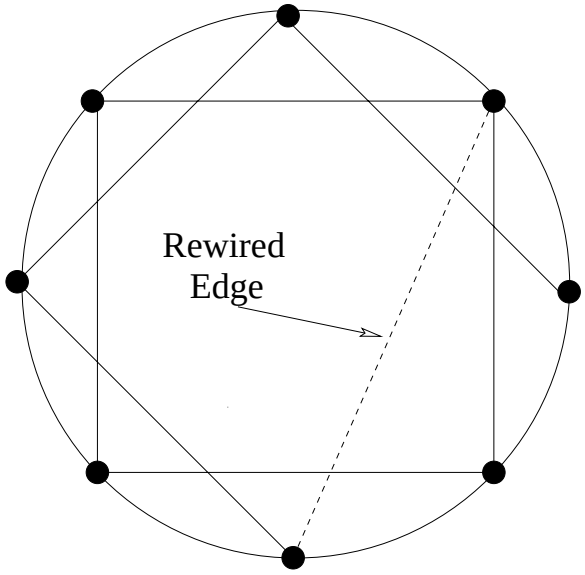

Fig. 8. Ring lattice with shortcuts.

The second analysis is by Yang and Kornfeld [65]. They compare the performance of a hub and spoke network to other competing networks for overnight package delivery. The authors had

planned to test their model on 26 major cities but found that the resulting mixed-integer program was too large to solve (about 2,000 integer variables and 17,000 non-integer variables). Consequently, they focused on two subsets of seven cities—the first seven cities in the alphabetical list and the largest seven cities in the list. The optimal solution for the first seven cities is one hub in Atlanta and three partial hubs. For the seven largest cities (with respect to cargo) the solution contains one hub in New York City and four partial hubs. Next the authors examined 5 cities placed on a circle. By varying the demand pattern a variety of network configurations proved optimal. We note that it should be possible to solve (or nearly solve) much larger networks for the Yang and Kornfeld model. Column generation techniques for large $0-1$ integer linear programs have proven quite effective in generating high quality solutions when over a million decision variable are possible. For example, Barnhart and Schneur [10] solve a network design problem (hub and spoke) for an express mail carrier in which the linear programming relaxation had $800,000$ columns of which $20,000$ were chosen and passed on to the integer programming solver. The resulting gap between the solution generated by this approach and the true optimal solution was shown to be less than 0.5 percent.

In contrast to these computational studies there are standard combinatorial problems whose study may also bring some light to the issues in designing an airline routing network for general aviation. In particular, Korte and Vygen [46] devote a chapter to network design problems in their recent research monograph. In this chapter the survivable network design problem is of interest. It is posed as follows. We are given an undirected graph G with edge weights and connectivity requirements $r(i,j)$ for all pairs of nodes i and j. The goal is to find a minimum weight spanning subgraph of G such that for every pair of nodes i and j there exist $r(i,j)$ edge-disjoint paths from i to j in the subgraph. The $r(i,j)$ values provide a specified measure of redundant connections between the nodes in the graph, thus protecting the graph from edge failures. As expected the survivable network design problem is NP-hard (the Steiner tree problem is a special case).

The general aviation network design problem will share some similarities with the overnight package delivery model of Yang and Kornfeld [65]. That is, we expect there will not be regular planned routes and connections between origin and destination pairs. Instead, each day (or several times a day) demands for service will be collected and a route network will be constructed. Ideally the routing would be from door to door, not simply from airport to airport. There are many more questions at this point than answers. If we are constructing the network via an optimization approach, what are the objectives and constraints? The previously mentioned studies both fail to consider large enough sets of nodes to know whether scale-free graphs are a viable alternative. (A maximum of 24 for Lederer and Nambimadom [47] and only 7 for Yang and Kornfeld [65]).

We close with Table 2, a broad category summary of our literature review of scale-free networks and general aviation.

Category	**References**
surveys/books	[53, 24, 3, 60, 17, 64, 8]
optimization	[59, 31, 19, 49, 50, 33, 61]
construction	[7, 1, 25, 43, 62, 48, 45, 26, 14, 54]
dynamical systems	[28, 39]
robustness/fragility	[2, 21, 13, 22, 57, 18, 56]
algorithmic/mathematics	[44, 37, 23, 11, 12, 40, 52]
empirical results	[4, 6, 9, 55]
airline network design	[65, 47, 10, 38, 27, 16, 5]

Table 2. Reference categories for scale-free networks and general aviation.

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14. ABSTRACT

In the late 1990s a number of researchers noticed that networks in biology, sociology, and telecommunications exhibited similar characteristics unlike standard random networks. In particular, they found that the cummulative degree distributions of these graphs followed a power law rather than a binomial distribution and that their clustering coefficients tended to a nonzero constant as the number of nodes, n, became large rather than $O(1/n)$. Moreover, these networks shared an important property with traditional random graphs—as n becomes large the average shortest path length scales with log n. This latter property has been coined the small-world property. When taken together these three properties—small-world, power law, and constant clustering coefficient—describe what are now most commonly referred to as scale-free networks. Since 1997 at least six books and over 400 articles have been written about scale-free networks. In this manuscript an overview of the salient characteristics of scale-free networks. Computational experience will be provided for two mechanisms that grow (dynamic) scale-free graphs. Additional computational experience will be given for constructing (static) scale-free graphs via a tabu search optimization approach. Finally, a discussion of potential applications to general aviation networks is given.

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